工程統計-第五章

估計

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第五章 估計

- ■5-1 估計量與估計
- ■5-2 母體平均數的區間估計
- ■5-3 母體變異數的區間估計

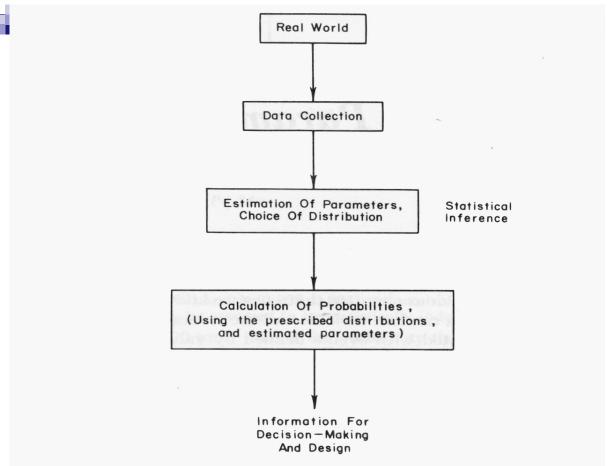
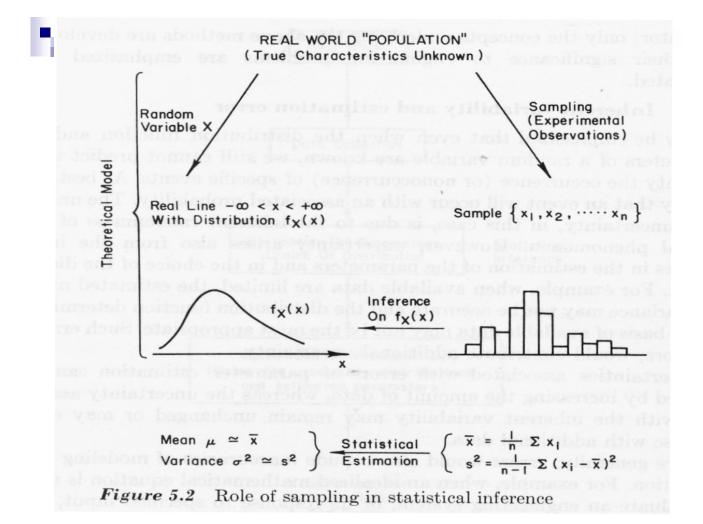


Figure 5.1 Role of statistical inference in decision-making process





5-1 估計量與估計

- 1、由母體抽出的特性與統計分析的結果,來 推估未知母體參數真實值的方法及過程,稱 為估計(Estimate)。
- 2、推估母體參數的樣本統計量(如平均數、 樣本比例或變異數),稱為**估計量** (Estimator)。

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依據研究問題特性,估計方法有

- 1. 點估計; 2. 區間估計
 - *例如,某等候線之平均等候時間可以表為區間估計

(Interval Estimate)

$$35 \le \mu \le 45$$
秒

由於 μ 並非某一確定值,故以區間可能的範圍表示即 [35,45]。區間估計亦可表示為:

$$\mu = 40 \pm 5$$

比較準確平均等候時間估計方式

$$\mu = 40\pm 1$$

- y.
 - This results must *not* be interpreted as:

 The value μ lies with probability 0.9 between 35 and 40. Such a probability statement can't be correct, because no random variable is involved.
 - The statement "The random interval [L,U] contains with probability 0.9 " is correct.

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精確度與可靠度

- 統計估計的品質好壞可以用精確度 (Precision)與可靠度(Reliability)來評量。
- 精確度高低是指估計量的誤差(即區間)大小。
- **可靠度**高低是指正確估計量的發生機率的大小。
- ■點估計(Point Estimate)是根據樣本資料所 求得的單一的統計量。



- 一個好的估計同時考慮精確度與可靠度
- 精確度高而可靠度低(機率非常小)的估計結果39 ≤ µ ≤ 41秒
- 可靠度高(機率非常高)但精確度低的估計結果20 ≤ μ ≤ 60 秒

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◎最適估計量之選取準則

- *估計的目的在於尋找可作為母體參數值最 佳或最適當的估計量。
- *常用的判斷準則為

不偏性、

一致性、

有效性。



不偏估計量(Unbiased Estimator)

■當估計量之抽樣分配的期望值等於母體參數值時,稱之為不偏性(Unbiased),而具有不偏性的估計量,是一不偏估計量(Unbiased Estimator);反之,則稱為偏估計量(biased Estimator)。

樣本 為母體μ的不偏估計量,即:

$$E(\overline{X}) = \mu$$

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一致估計量(Consistent Estimators)

若一不偏估計量隨著樣本數的增加而愈接近母體參數,則稱此不偏估計量具有一致性。此特性可由抽樣分配之標準誤得知 \overline{X} 是 μ 的一致估計量,因為當n 趨大時, \overline{X} 抽樣分配的變異量就愈小,而使餐接近 μ 的 \overline{Y} 增加。



◎有效估計量(Efficient Estimators)

由於一母體往往有數個不偏估計量(比如 \overline{X} 與中位數皆為 μ 之不偏估計量),若欲選擇其中最佳的估計量,則需要有效性作為判斷:

在二個不偏估計量中,具有較小變異數 (即有較高的精確度與可靠度)者,稱為 較有效的估計量。

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5-2 母體平均數的區間估計

- *點估計的結果比區間估計較為不準確。
- *由於樣本 \overline{X} 之抽樣分配通常以常態分配來近似,可以 \overline{X} 落於某特定區間的機率來表示,故區間估計亦較點估計來得適宜。
- *如何決定區間之左右兩個端點之值呢?
- *例如,0.95的機率區間係為由X為中心與兩端點(界限)值 $\overline{X}\pm 1.96\sigma_{\overline{X}}$ 所構成的信賴區間,即

$$\Pr[\overline{X} - 1.96\sigma_{\overline{X}} \le \mu \le \overline{X} + 1.96\sigma_{\overline{X}}] = 0.95$$



 \overline{X} 在信賴水準 $1-\alpha$ 下,其常態曲線由兩端點值所包含的面積亦為 $1-\alpha$,而不在此區間的左右兩端面積和為 α ,故每單邊剩餘面積為 $\alpha/2$ 。故

$$\mu = \overline{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

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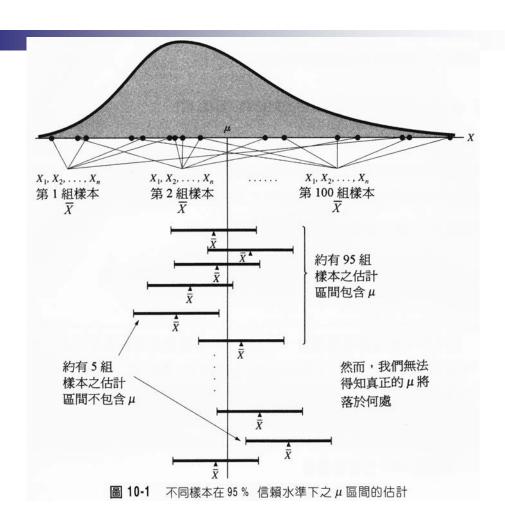
*母體平均數 μ 的 (1-lpha)100%的信賴區間,可用下式來

表達:

$$\mu = \overline{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \overline{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \le \mu \le \overline{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

信賴水準或稱信心水準(Confidence Level),係指所產生的區間估計值,可真正包含被估計母體參數的比例(機率)。

信賴:是因為機率值表示了計算之區間包含母體參數的可能性,亦即區間估計機率愈高,其包含母體參數的信賴度就愈高。



當母體 σ 已知,其信賴區間的計算式為:

$$\mu = \overline{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \not \propto \overline{X} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \le \mu \le \overline{X} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

◎σ未知時之母體信賴區間的估計 樣本夠大(n>30)適合常態分布 小樣本應用t分配比較適當

其母體 μ 之 $100(1-\alpha)%$ 信賴區間為

$$\mu = \overline{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \quad \text{ is } \quad \overline{X} - t_{\alpha/2} \frac{s}{\sqrt{n}} \leq \mu \leq \overline{X} + t_{\alpha/2} \frac{s}{\sqrt{n}}$$

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Table G Student t Distribution



The following table provides the values of t_{α} that correspond to a given upper-tail area α and a specified number of degrees of freedom.

Upper-Tail Area α									
.4	.25	.1	.05	.025	.01	.005	.0025	.001	.000
.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
.277	.765	1.638	2.353	3.182		5.841	7.453	10.214	12.92
.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
.261	.703	1.383	1.833	2.262	2.821	3.250	3,690	4.297	4.781
.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.22
.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
									4.015
									3.965
.257									3.922
.257	.688			2.093					3.883
257	687			2.086					3.850
									3.819
									3.792
									3.76
									3.74
									3.725
									3.70
									3.690
									3.67
									3.659
									3.64
									3.55
									3.46
									3.37
.253	.674	1.289	1.645	1.960	2.338	2.576	2.807	3.160	
	.325 .289 .277 .271 .265 .263 .262 .261 .260 .259 .258 .258 .257 .257 .257 .257 .257 .256 .256 .256 .256 .256 .256 .256 .256	.325 1.000 .289 816 .277 .765 .271 .741 .267 .727 .265 .718 .263 .711 .262 .706 .261 .703 .260 .700 .260 .697 .259 .695 .258 .699 .258 .699 .258 .699 .257 .689 .257 .689 .257 .688 .257 .687 .257 .686 .256 .686 .256 .685 .256 .684 .256 .684 .256 .684 .256 .683 .256 .683 .256 .683 .256 .683 .256 .683 .256 .683	325 1.000 3.078 289 .816 1.886 277 .765 1.638 271 .765 1.638 271 .741 1.533 267 .727 1.476 263 .711 1.415 262 .706 1.397 261 .703 1.383 260 .700 1.375 260 .697 1.363 259 .695 1.356 259 .694 1.350 258 .699 1.337 257 .689 1.337 257 .689 1.330 257 .688 1.328 257 .687 1.325 257 .686 1.325 257 .687 1.325 257 .688 1.330 257 .688 1.318 256 .684 1.316 256 .684 1.316 256 .684 1.316 256 .684 1.316 256 .683 1.311 256 .683 1.311 256 .683 1.311 256 .683 1.311 256 .683 1.311	.4 .25 .1 .05 .325 1.000 3.078 6.314 .289 .816 1.886 2.920 .277 .765 1.638 2.353 .271 .741 1.533 2.132 .267 .727 1.476 2.015 .265 .718 1.440 1.943 .263 .711 1.415 1.895 .262 .706 1.397 1.860 .261 .703 1.383 1.833 .260 .700 1.372 1.812 .260 .697 1.363 1.796 .259 .695 1.356 1.782 .259 .694 1.350 1.771 .258 .692 1.345 1.761 .258 .690 1.337 1.746 .257 .688 1.330 1.746 .257 .688 1.330 1.734 .257 .688 1.328 1.725 .257 .688 1.325 1.725 .257 .686 1.325 1.725 .257 .686 1.321 1.717 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .684 1.316 1.708 .256 .683 1.311 1.699 .256 .683 1.311 1.699 .256 .683 1.311 1.699 .256 .683 1.311 1.699 .256 .683 1.310 1.697 .255 .681 1.303 1.684 .254 .677 1.289 1.558	.4 .25 .1 .05 .025 .325 1.000 3.078 6.314 12.706 .289 .816 1.886 2.920 4.303 .277 .765 1.638 2.353 3.182 .271 .741 1.533 2.132 2.776 .267 .727 1.476 2.015 2.571 .265 .718 1.440 1.943 2.447 .263 .711 1.415 1.895 2.365 .262 .706 1.397 1.860 2.306 .261 .703 1.383 1.833 2.262 .261 .703 1.383 1.833 2.262 .260 .700 1.372 1.812 2.228 .260 .697 1.365 1.782 2.179 .259 .694 1.350 1.771 2.160 .258 .692 1.345 1.761 2.145 .258 .690 1.337 1.746 2.120 .257 .688 1.330 1.734 2.101 .257 .688 1.330 1.734 2.101 .257 .688 1.330 1.734 2.101 .257 .688 1.328 1.729 2.093 .257 .686 1.321 1.725 2.086 .255 .686 1.321 1.717 2.064 .256 .686 1.321 1.717 2.074 .256 .686 1.321 1.717 2.074 .256 .686 1.321 1.717 2.074 .256 .684 1.316 1.708 2.060 .256 .684 1.316 1.708 2.060 .256 .684 1.316 1.708 2.050 .256 .684 1.316 1.708 2.050 .256 .684 1.316 1.708 2.056 .256 .684 1.316 1.708 2.056 .256 .684 1.311 1.700 2.052 .256 .683 1.310 1.697 2.042 .255 .681 1.303 1.697 2.042 .255 .681 1.303 1.697 2.042 .255 .683 1.310 1.697 2.042 .255 .681 1.303 1.684 2.021 .255 .681 1.303 1.684 2.021 .255 .681 1.303 1.684 2.021 .255 .681 1.303 1.684 2.021	.4 .25 .1 .05 .025 .01 .325 1.000 3.078 6.314 12.706 31.821 .289 .816 1.886 2.920 4.303 6.965 .277 .765 1.638 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3.355 3.883 4.501 .261 .703 1.383 1.833 2.262 2.821 3.250 3.690 4.297 .260 .700 1.372 1.812 2.228 2.764 3.169 3.355 3.833 4.501 .261 .703 1.383 1.833 2.262 2.821 3.250 3.690 4.297 .260 .700 1.372 1.812 2.228 2.764 3.169 3.497 4.025 .259 .695 1.356 1.782 2.179 2.681 3.055 3.428 3.930 .259 .694 1.350 1.771 2.160 2.650 3.012 3.372 3.852 .258 .692 1.345 1.761 2.145 2.624 2.977 3.326 3.787 .258 .691 1.341 1.753 2.131 2.602 2.947 3.286 3.783 .258 .690 1.337 1.746 2.100 2.583 2.921 3.252 3.686 .257 .688 1.330 1.734 2.101 2.567 2.898 3.222 3.646 .257 .688 1.330 1.734 2.101 2.552 2.878 3.197 3.610 .257 .688 1.330 1.734 2.101 2.552 2.878 3.197 3.610 .257 .688 1.328 1.729 2.093 2.539 2.861 3.174 3.579 .257 .688 1.328 1.729 2.093 2.539 2.861 3.174 3.579 .257 .688 1.323 1.725 2.086 2.528 2.845 3.153 3.552 .257 .686 1.323 1.721 2.086 2.528 2.845 3.153 3.552 .257 .686 1.323 1.721 2.086 2.528 2.845 3.153 3.552 .257 .686 1.323 1.721 2.080 2.518 2.831 3.135 3.527 .256 .685 1.319 1.714 2.069 2.500 2.807 3.104 3.485 .256 .684 1.316 1.708 2.060 2.485 2.777 3.091 3.467 .256 .684 1.316 1.708 2.060 2.485 2.777 3.091 3.467 .256 .684 1.311 1.699 2.045 2.462 2.756 3.038 3.396 .256 .683 1.311 1.699 2.045 2.462 2.756 3.038 3.396 .256 .683 1.311 1.699 2.045 2.462 2.756 3.038 3.396 .256 .683 1.310 1.697 2.042 2.457 2.779 3.007 3.030 3.385 .256 .684 1.316 1.708 2.060 2.485 2.770 3.030 3.385 .256 .684 1.316 1.709 2.042 2.457 2.779 3.007 3.421 .256 .683 1.311 1.699 2.045 2.462 2.756 3.038 3.396 .256 .684 1.316 1.709 2.042 2.457 2.770 3.030 3.385 .256 .683 1.310 1.697 2.042 2.457 2.770 3.030 3.385 .256 .683 1.310

Source: E. S. Pearson and H. O. Hartley, Biometrika Tables for Statisticians. Vol. I. London: Cambridge University Press, 1966. Partly derived from Table III of Fisher and Yates, Statistical Tables for Biological, Agricultural, and Medical Research, published by Longman Group Ltd., London (previously published by Oliver & Boyd, Edinbugh, 1963). Reproduced with permission of the authors and publishers.





表 10-1 同位素之實驗結果

化學元素量	同位素量	同位素百分比
(mg)	(mg)	(%)
1.5	0.160	10.7
1.5	0.260	17.3
1.0	0.135	13.5
1.5	0.204	13.6
1.5	0.079	5.3
3.0	1.140	38.0
3.0	0.540	18.0
1.2	0.235	19.6
3.1	0.637	20.5
3.3	0.420 $\overline{X} = 16.92$	12.7
	$\Lambda = 10.92$	s = 0.072



◎決定所需的樣本大小

在 $(1-\alpha)100\%$ 信賴區間及準確度 d 要求下,所須樣本

大小的計算式為
$$n = \frac{z_{\alpha/2}^2 \sigma^2}{d^2}$$

其中

d=所需的準確度(或估計最大誤差)。

 $Z_{\alpha/2}$ =可靠度為 $1-\alpha$ 之標準常態分配的臨界值。

σ=假設的母體標準差。

可靠度定義為

可靠度 =
$$\Pr[|X - \mu| \le d]$$

20

5-3 母體變異數的區間估計

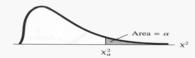
- *由於資料的變異數大小可作為衡量平均數是否具有 代表性的指標,母體變異數的估計量的應用亦相當 重要。
- * $(1-\alpha)100\%$ 之 σ^2 的信賴區間:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

卡方分配為一非對稱的曲線,因此由表中查得自由度n-1的兩個臨界值 $\chi^2_{\alpha/2}$ 與 $\chi^2_{1-\alpha/2}$



Table H Chi-Square Distribution



The following table provides the values of χ^2_{α} that correspond to a given upper-tail area α and a specified number of degrees of freedom.

Degrees	Upper-tail Area $lpha$								
Freedom	.99	.98	.95	.90	.80	.70	.50		
1	.000157	.000628	.00393	.0158	.0642	.148	.455		
2	.0201	.0404	.103	.211	.446	.713	1.386		
3	.115	.185	.352	.584	1.005	1.424	2.366		
4	.297	.429	.711	1.064	1.649	2.195	3.357		
5	.554	.752	1.145	1.610	2.343	3.000	4.351		
6	.872	1.134	1.635	2.204	3.070	3.828	5.348		
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346		
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344		
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343		
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342		
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341		
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340		
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340		
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339		
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339		
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338		
17	6.408	7.255	8.672	10.085	12.002	13.531	16.338		
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338		
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338		
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337		
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337		
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337		
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337		
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337		
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337		
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336		
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336		
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336		
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336		
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336		